

Signatures of quantum gravity in gravitational-wave observables

Gianluca Calcagni

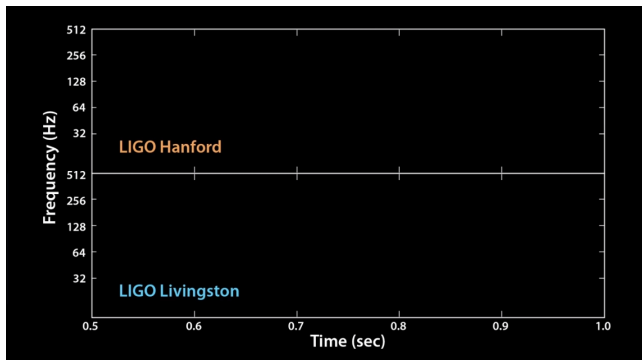
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01/21– Gravitational waves (GWs)

Abbott et al. 2016



Theories **beyond Einstein**: any imprint in GW **production** or **propagation**?

02/21– QG modifications of propagation and generation

From **modified dispersion relations**:

- Propagation speed*.
- Waveform phase [Mirshekari et al. 2012].
- Luminosity distance*.
- EMRIs kludge waveform?

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From **dimensional flow**:

- All the above, with added ingredients*.

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From **dimensional flow**:

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From **primordial blue-tilted spectra**:

- Stochastic GW background.

03/21– Dispersion relation and propagation speed

Single GW events can place bounds on the **propagation speed** of gravitons [Ellis et al. 2016; Arzano & G.C. PRD 2016; Yunes et al. 2016], on violations of the equivalence principle and of Lorentz invariance in theories beyond Einstein [Yunes et al. 2016].

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Modified dispersion relation for the graviton ($\Delta v = |d\omega/dk - 1|$ group velocity) [Arzano & G.C. PRD 2016]

$$\omega^2 = k^2 \left(1 \pm \frac{k^n}{M^n} \right) + O(k^{n+3}) \quad \Rightarrow \quad M \simeq \frac{\omega}{\Delta v^{\frac{1}{n}}}$$

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→ LISA: $\omega_{\text{LISA}}/\omega_{\text{LIGO}} \sim 10^{-5}$, about same constraint level on M if $|\Delta v| < 10^{-20-5n}$

04/21– Bounds

Ellis et al. MPLA 2016; Arzano & G.C. PRD 2016; G.C. EPJC 2017, JHEP 2017

Recovery of the entropy-area law [Padmanabhan 1997,1998]:

$$M > 6 \times 10^{-3} \text{ eV}, \quad n = 2$$

Generic quantum-gravity/stringy arguments [Amelino-Camelia et al. 1997]:

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This range is typical of field theories on **multifractal** geometries [G.C. 2012-2017], where $n = 1 - d_H/4$ is related to the UV Hausdorff dimension of spacetime. For the typical $d_H = 2$,

$$M > 10^{17} \text{ GeV}, \quad n = 0.5$$

05/21– Standard sirens: optical sources of GWs

EM luminosity distance: Flux = power per unit area

$$F =: \frac{L}{4\pi(d_L^{\text{EM}})^2}, \quad d_L^{\text{EM}} = \frac{a_0^2}{a} r = (1+z) \int_0^z \frac{dz'}{H(z')}$$

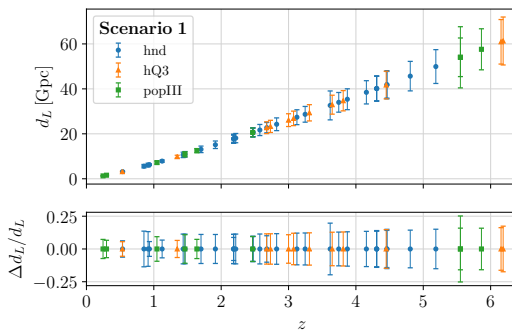
GW luminosity distance:

$$h \propto \frac{1}{d_L^{\text{GW}}} \stackrel{\text{in GR}}{=} \frac{1}{d_L^{\text{EM}}}$$

Known example (LIGO-Virgo/Fermi): BNS GW170817 /
GRB170817A [Abbott et al. 2017]

06/21– Standard sirens @ LISA

Belgacem, G.C., et al. [CosWG] JCAP 2019

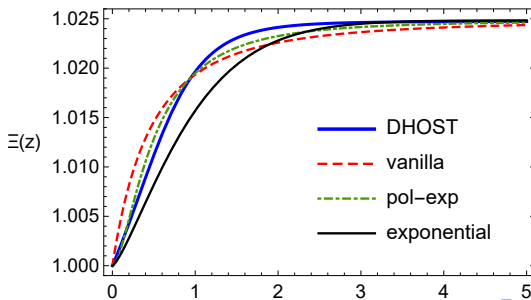


07/21– Parametrizing $d_L(z)$: model selection

Belgacem, G.C., et al. [CosWG] JCAP 2019

$$\frac{d_L^{\text{GW}}}{d_L^{\text{EM}}} = \Xi_0 + \frac{1 - \Xi_0}{(1+z)^n}, \quad \Xi_0 + \frac{1 - \Xi_0}{1+z^m}, \quad \Xi_0 + (1 - \Xi_0) e^{-z^n}$$

Checked with DHOST, bigravity, IR nonlocal gravity, QG, ...



08/21– Quantum gravity (QG)

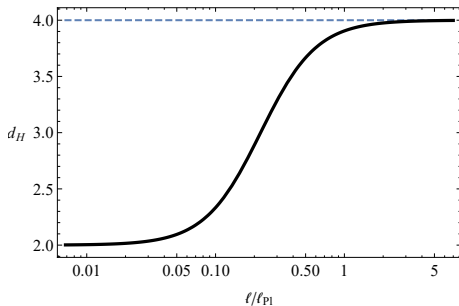
- **Perturbative QG**: effects important at high curvature/short distances. Homogeneous cosmology, only ℓ_{Pl} and H available, corrections $(\ell_{\text{Pl}}H)^n \stackrel{\text{today}}{\sim} (10^{-60})^n$, $n = 1, 2, 3, \dots$
- **Non-perturbative QG effects**, e.g., models with a **third scale** $L \gg \ell_{\text{Pl}}$, quantum corrections $\sim \ell_{\text{Pl}}^a H^b L^c$ with $a - b + c = 0$, not all of them small [Bojowald, G.C. & Tsujikawa, PRL 2011].



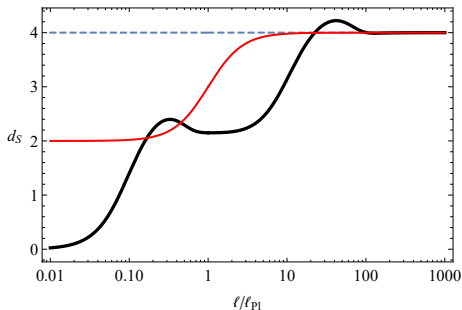
10/21– Dimensional flow

Universal **non-perturbative** effect in QG [’t Hooft 1993; Carlip 2009; G.C. PRL 2010; Carlip 2017]: running **Hausdorff** and **spectral** dimensions.

d_H : scaling of volume



d_S : scaling of dispersion rel.



11/21– Luminosity distance in QG: 1

G.C. et al. JCAP 2019; PLB 2019

$$S = \frac{1}{2} \int d\varrho(x) h_{ij} \mathcal{K}(\square) h^{ij}, \quad [h_{ij}] = \frac{d_H - [\mathcal{K}]}{2} =: \mathbf{\Gamma}$$

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Modified dispersion relation, e.g., $\mathcal{K}(-k^2) = -\ell_*^{2-2\beta} k^2 + k^{2\beta}$.

Return probability and spectral dimension

$$\mathcal{P}(\sigma) \propto \int d\tilde{\varrho}(k) e^{-\sigma \ell_*^{[\mathcal{K}]} \mathcal{K}(-k^2)}, \quad d_S := -2 \frac{d \ln \mathcal{P}(\sigma)}{d \ln \sigma} = 2 \frac{d_H^k}{[\mathcal{K}]}$$

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$$\mathbf{\Gamma} = \frac{d_{\text{H}}}{2} - \frac{d_{\text{H}}^k}{d_{\text{S}}}, \quad d_{\text{S}} \neq 0$$

12/21– Luminosity distance in QG: 2

G.C. et al. JCAP 2019; PLB 2019

Local wave zone approximation $\omega r \gg 1$:

$$h_{ij} = \frac{\kappa \mathcal{F}_{ij}(t-r)}{(r^2)^{\frac{\Gamma}{2}}} \sim \frac{1}{r^\Gamma} \rightarrow \frac{1}{(d_L^{\text{EM}})^\Gamma}$$

Cosmological propagation (flat FLRW): $r \rightarrow ar$ rule from covariance:

$$h \propto \frac{1}{(d_L^{\text{EM}})^\Gamma} \quad \Rightarrow \quad h^{\text{UV}} \simeq \frac{1}{(d_L^{\text{EM}})^{\Gamma_{\text{UV}}}}, \quad h^{\text{IR}} \simeq \frac{1}{d_L^{\text{EM}}}$$

13/21– Master formula

G.C. et al. JCAP 2019; PLB 2019

$$\frac{d_L^{\text{GW}}}{d_L^{\text{EM}}} = 1 \pm |\gamma - 1| \left(\frac{d_L^{\text{EM}}}{\ell_*} \right)^{\gamma-1}$$

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$$\frac{d_L^{\text{GW}}}{d_L^{\text{EM}}} = 1 \pm |\gamma - 1| \left(\frac{d_L^{\text{EM}}}{\ell_*} \right)^{\gamma-1}$$

Very similar to models with extra dimensions [Deffayet & Menou 2007;
Pardo et al. 2018; Abbott et al. 2018]

$$\frac{d_L^{\text{GW}}}{d_L^{\text{EM}}} = \left[1 + \left(\frac{d_L^{\text{EM}}}{R_c} \right)^{n_c} \right]^{\frac{D-4}{2n_c}}$$

14/21– UV geometry in QG (unobservable)

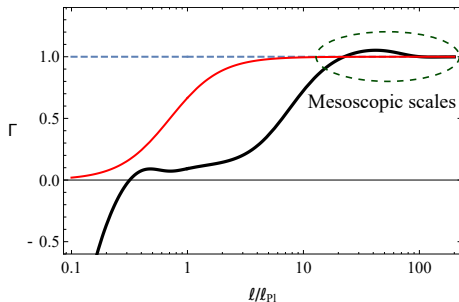
G.C. et al. JCAP 2019

	d_H^{UV}	$d_H^{k,\text{UV}}$	d_S^{UV}	Γ_{UV}	$\Gamma_{\text{meso}} \gtrsim 1$
GFT/spin foams/LQG	2	4	$[1, 4)$	$[-3, 0)$	✓
Causal dynamical triangulations (phase C)	4	4	$3/2$	$-2/3$	
κ -Minkowski bicovariant ∇^2 (c.i.m.)	1	3	3	$-1/2$	
κ -Minkowski bicross-product ∇^2 (c.i.m.)	1	3	6	0	
Stelle gravity	4	4	2	0	
String theory (low-energy limit)	D	D	2	0	
Asymptotic safety	4	4	2	0	
Hořava–Lifshitz gravity	4	4	2	0	
κ -Minkowski relative-locality ∇^2 (c.i.m.)	1	3	$+\infty$	$1/2$	
κ -Minkowski bicovariant ∇^2 (o.m.)	4	3	3	1	
κ -Minkowski bicross-product ∇^2 (o.m.)	4	3	6	$3/2$	✓
κ -Minkowski relative-locality ∇^2 (o.m.)	4	3	$+\infty$	2	✓
Padmanabhan's non-local model	4	4	$+\infty$	2	✓

15/21– QG and standard sirens. 1

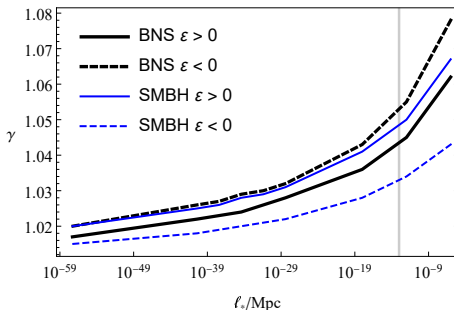
G.C. et al. JCAP 2019

Standard sirens: **BNS** GW170817/GRB 170817A (LIGO-Virgo public data) and simulated **SMBH**, $d_L^{\text{EM}} = 15.96$ Gpc, $z = 2$ (LISA catalogs). Detectable QG effect if $\gamma \gtrsim 1$, even when $\ell_* = O(\ell_{\text{Pl}})$:



16/21– QG and standard sirens. 2

G.C. et al. PLB 2019



$$\Gamma_{\text{meso}} - 1 < 0.02$$

17/21– QG and standard sirens. 3

G.C. et al. JCAP 2019

Theories where $\gamma = \Gamma_{\text{meso}} \gtrsim 1$:

- Non-commutative κ -Minkowski spacetime

$$\Gamma_{\text{meso}} \simeq 1 + \frac{5}{96\pi} \frac{\ell_{\text{Pl}}^2}{\ell^2} \sim 1 + 10^{-120}.$$

- Padmanabhan's model near BH horizon

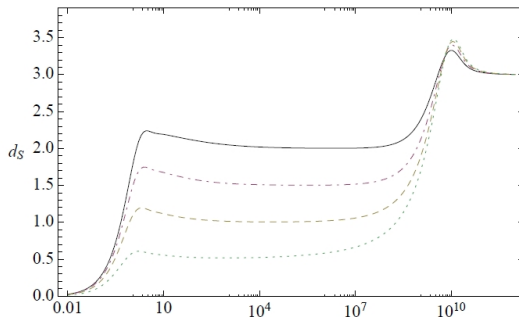
$$\Gamma_{\text{meso}} \simeq 1 + \frac{5\pi}{2} \frac{\ell_{\text{Pl}}^2}{\ell^2} \sim 1 + 10^{-120}.$$

- QGs with discrete pre-geometries: GFT, spin foams, LQG.

Γ_{meso} **strongly state dependent.**

18/21– Future work: testing GFT/spin foams/LQG

Numerical analysis of dimensional flow [G.C., Oriti & Thürigen
2013,2014,2015]



Γ_{meso} can be calculated from realistic quantum states of geometry.

19/21– Nonlocal quantum gravity

Briscese, G.C. & Modesto PRD 2019

Linearized perturbation equation:

$$\square \tilde{h} = 0, \quad \tilde{h} = e^{H(\square)} h, \quad H(\square) = -\ell_*^2 \square$$

Luminosity distance:

$$\tilde{h} \propto \frac{1}{d_L^{\text{GW}}} \quad \Rightarrow \quad h \propto e^{-H} \frac{1}{d_L^{\text{GW}}}.$$

$$\frac{d_L^{\text{GW}}}{d_L^{\text{EM}}} \simeq 1 + c(\ell_* H_0)^2 \sim 1 + 10^{-120}, \quad c = O(1) - O(10)$$

Unobservable, sorry!

20/21– Strain noise and QG

Amelino-Camelia 1998, 2013; Amelino-Camelia et al. PLB 2017, G.C. et al. JCAP 2019

What if QG spacetime uncertainty (fuzziness) \sim strain noise?

$$\left(\frac{\ell_*}{L}\right)^{2(1-\alpha)} = \frac{\sigma_{\text{QG}}^2}{L^2} \sim \sigma_{\text{exp}}^2 \simeq f \mathcal{S}^2(f) \Big|_{f=\frac{c}{L}}.$$

Uncertainty/fuzziness at Planck scales $\ell_* = \ell_{\text{Pl}}$ (intrinsic QG noise) no greater than the strain noise:

	\mathcal{S} (Hz $^{-1/2}$)	f (Hz)	α
LIGO/Virgo/KAGRA	10^{-23}	10^2	< 0.47
LISA	10^{-20}	10^{-2}	< 0.54
DECIGO	10^{-23}	10^{-1}	< 0.47

Model-independent bound in the UV:

$$d_{\mathrm{H}}^{\mathrm{UV}} = 4\alpha < 1.9.$$

21/21– What to bring home

- We can and should ask more from our theories and push them to get phenomenology in GW astronomy (even negative one).
- Better to make this effort in fundamental theories (strings, quantum gravities, etc.) rather than in ad-hoc models.

ご清聴ありがとうございました

Eskerrik asko

Thank you

Muchas gracias

Grazie

Muito obrigado

Kiitos paljon

Danke schön

Merçi beaucoup

Спасибо